Remnant of binary black-hole mergers: The spinless case revisited

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We developed phenomenological expansions based on a (4\textsuperscript{th} order) Taylor expansion in terms of variables $q, S_1, S_2$ but restricted to Parity and exchange $1\leftrightarrow 2$ symmetries.

• 61 New aligned spin simulations + 10 new nonspinning
• $1/3 \leq q \leq 1$ and $-0.85 \leq a_i \leq +0.85$ for the spinning cases
• $1/6 \leq q < 1$ for the nonspinning
• Runs give 8-10 orbits prior to merger at $e \approx 10^{-3}$

• This is in addition to the 36 runs we did in 2014

• Those two sets of runs will form the core of the new RIT waveform catalog soon to be public.


NR/LSC teams assembled more than a thousand NR waveforms, now fully integrated in LIGO Algorithm Library (LAL). This can be used to directly estimate parameters of BBHs from NR without the use of models: Abbot et al. arXiv:1606.01262.
The fitting formula for $M_{\text{rem}}$ is given by

$$
\frac{M_{\text{rem}}}{m} = (4\eta)^2 \{ M_0 + K_1 \tilde{S}_|| + K_2a \tilde{\Delta}_|| \delta m + K_2b \tilde{S}_||^2 \\
+ K_2c \tilde{\Delta}_||^2 + K_2d \delta m^2 + K_3a \tilde{\Delta}_|| \tilde{S}_|| \delta m + K_3b \tilde{S}_|| \tilde{\Delta}_||^2 \\
+ K_3c \tilde{S}_||^3 + K_3d \tilde{S}_|| \delta m^2 + K_4a \tilde{\Delta}_|| \tilde{S}_||^2 \delta m + K_4b \tilde{\Delta}_||^3 \delta m \\
+ K_4c \tilde{\Delta}_||^4 + K_4d \tilde{S}_||^4 + K_4e \tilde{\Delta}_|| \tilde{S}_||^2 + K_4f \delta m^4 \\
+ K_4g \tilde{\Delta}_|| \delta m^3 + K_4h \tilde{\Delta}_||^2 \delta m^2 + K_4i \tilde{S}_||^2 \delta m^2 \} \\
+ [1 + \eta(\tilde{E}_{\text{isco}} + 11)] \delta m^6, \tag{1}
$$

where

$$
m = m_1 + m_2,
$$

$$
\delta m = \frac{m_1 - m_2}{m},
$$

$$
\tilde{S} = \frac{\tilde{S}_1 + \tilde{S}_2}{m^2},
$$

$$
\tilde{\Delta} = \frac{\tilde{S}_2/m_2 - \tilde{S}_1/m_1}{m},
$$

where $m_i$ is the mass of BH $i = 1, 2$ and $\tilde{S}_i$ is the spin of BH $i$. We also use the auxiliary variables

$$
\eta = \frac{m_1 m_2}{m^2},
$$

$$
q = \frac{m_1}{m_2},
$$

$$
\tilde{\alpha}_i = \frac{\tilde{S}_i}{m_i^2},
$$

where $|\tilde{\alpha}_i| \leq 1$ is the dimensionless spin of BH $i$, and we use the convention that $m_1 \leq m_2$ and hence $q \leq 1$. Here the index $\perp$ and $\parallel$ refer to components perpendicular to and parallel to the orbital angular angular momentum.
Note that the two formulas, for the final mass and final spin impose the particle limit through their ISCO contributions.

The use of final horizon measures for the final mass and spins greatly increases the accuracy (by 1-2 orders of magnitude) wrt radiation measures and is completely consistent with them.
We fit 19+19 coefficients above to 175 of ours and SXS simulations and compare the new (V2) to the old (V1) fits

Final mass and spin modeling for nonprecessing binaries are already very good!
We model the total recoil as

\[ \vec{V}_{\text{recoil}}(q, \vec{\alpha}_i) = v_m \hat{e}_1 + v_\perp (\cos(\xi) \hat{e}_1 + \sin(\xi) \hat{e}_2), \quad (4) \]

\( \hat{e}_1, \hat{e}_2 \) are orthogonal unit vectors in the orbital plane, and \( \xi \) measures the angle between the “unequal mass” and “spin” contributions to the recoil velocity in the orbital plane, and with

\[ v_\perp = H\eta^2 (\tilde{\Delta}_\parallel + H_{2a}\tilde{S}_\parallel \delta m + H_{2b}\tilde{\Delta}_\parallel \tilde{S}_\parallel + H_{3a}\tilde{\Delta}_\parallel^2 \delta m \]
\[ + H_{3b}\tilde{S}_\parallel^2 \delta m + H_{3c}\tilde{\Delta}_\parallel \tilde{S}_\parallel^2 + H_{3d}\tilde{\Delta}_\parallel^3 + H_{3e}\tilde{\Delta}_\parallel \delta m^2 \]
\[ + H_{4a}\tilde{S}_\parallel \tilde{\Delta}_\parallel^2 \delta m + H_{4b}\tilde{S}_\parallel^3 \delta m + H_{4c}\tilde{S}_\parallel \delta m^3 \]
\[ + H_{4d}\tilde{\Delta}_\parallel \tilde{S}_\parallel \delta m^2 + H_{4e}\tilde{\Delta}_\parallel \tilde{S}_\parallel^3 + H_{4f}\tilde{S}_\parallel \tilde{\Delta}_\parallel^3 ), \]

\[ \xi = a + b\tilde{S}_\parallel + c\delta m\tilde{\Delta}_\parallel, \quad (5) \]

where

\[ v_m = \eta^2 \delta m(A + B\delta m^2), \quad (6) \]
We propose

\[ L_{\text{peak}} = (4\eta)^2 \left\{ N_0 + N_1 S_\parallel + N_{2a} \tilde{\Delta}_\parallel \delta_m + N_{2b} S_\parallel^2 \right. \]

\[ + N_{2c} \tilde{\Delta}_\parallel^2 + N_{2d} \delta m^2 + N_{3a} \tilde{\Delta}_\parallel S_\parallel \delta m + N_{3b} S_\parallel \tilde{\Delta}_\parallel^2 \]

\[ + N_{3c} S_\parallel^3 + N_{3d} S_\parallel \delta m^2 + N_{4a} \tilde{\Delta}_\parallel S_\parallel^2 \delta m + N_{4b} \tilde{\Delta}_\parallel^3 \delta m \]

\[ + N_{4c} \tilde{\Delta}_\parallel^4 + N_{4d} S_\parallel^4 + N_{4e} \tilde{\Delta}_\parallel^2 S_\parallel^2 + N_{4f} \delta m^4 \]

\[ + N_{4g} \tilde{\Delta}_\parallel \delta m^3 + N_{4h} \tilde{\Delta}_\parallel^2 \delta m^2 + N_{4i} S_\parallel^2 \delta m^2 \}. \quad (3) \]

Similar expansions can be carried out for the peak strain \( h \), News, \( \Psi_4 \), and their corresponding peak frequencies \( \omega_{\text{peak}} \).
We fit 17+19 coefficients above to ours 107 simulations and compare the new (V2) to the old (V1) fits.

These represents corrections of a few percent since they are measured directly from radiation waveforms. If one wants to improve upon this accuracy, one needs higher resolution simulations.
We have fitting formulae to predict the final properties of the merger of two black holes:

- The errors in the final mass and spin are \( \approx 0.1 - 0.2\% \)
- The errors in the recoil velocity and peak luminosity are \( \approx 5\% \)

The fittings produce a maximum radiated energy of above 11.3%:

\[
M_{\text{rem}}(1,1,1)=0.8867, \quad \text{while} \quad M(1,-1,-1)=0.968
\]

For equal mass binaries \( \alpha_f(1,1,1)=0.951, \alpha_f(1,-1,-1)=0.358 \)

The maximum recoil occurs for \( q \approx 2/3 \) at \( V_{\text{max}}(2/3,+1,-1)=516 \text{ km/s} \)

Those studies made use of models up to \( l \leq 6 \), extrapolation to infinite observer, and weighted the fitting from the case studies of three finite difference resolutions.

These well calibrated aligned spin formula are used for the nonprecessing extension.

One can also look at hints of resummation by Padé approximants (In fact already alternatively used for modeling recoils and final masses).
The nonspinning BBH simulations revisited

- 14 nonspinning BBH simulations
- $1/100 \leq q = m_1/m_2 \leq 1$
- 3 resolutions for Richardson extrapolation
  - N100, N120, N140 for $q \leq 1/6$
  - N100, N144, and N173 for $q = 1 = 10$
  - N144, N173, and N207 for $q = 1 = 15$
  - N100, N144, and N207 for $q = 1 = 100$.
- Perturbative extrapolation to infinite observer location
- Added up to $l \leq 6$ modes

To improve on these 3 main sources of error

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Expansions and particle limit in some detail

\[ M_{\text{final}} / M = M_0 + M_2 \delta m^2 + M_4 \delta m^4 + O(\delta m^6), \]  
with \( M_0, M_2, M_4 \), fitting parameters

But we know that in the particle limit

- \( m_{\text{final}} = m_{\text{initial}} = M \)
- \( \eta (E_{\text{ISCO}} / M - 1) \) is the energy radiated from \( \infty \)
- Comparable mass radiative terms \( E_{\text{rad}} \sim \eta^2 \)

In the particle limit \( M_{\text{final}} / M = 1 \) and \( \delta m^2 \to 1 \) and by adding a \( \delta m^6 \)-term we can impose this condition

\[ M_{\text{final}} / M = M_0 + M_2 \delta m^2 + M_4 \delta m^4 + M_6 \delta m^6 \]

to determine \( M_6 = 1 - M_0 - M_2 - M_4 \)

what allow to rewrite

\[ M_{\text{final}} / M = (1 - \delta m^2)(M_0 + M_2 \delta m^2 + M_4 \delta m^4) + \delta m^6 \]  
with \( (1 - \delta m^2) = 4\eta \)

Applying the same technique, now for the linear term in \( \eta \) we can impose \( \eta (E_{\text{ISCO}} / m - 1) \)

And obtain \( M_{\text{final}} / M = (4\eta)^2 (M_0 + M_2 \delta m^2 + M_4 \delta m^4) [1 + \eta (E_{\text{ISCO}} / m + 11)] \delta m^6 \)

A similar reasoning for the final spin leads to

\[ S_{\text{final}} / M_{\text{final}}^2 = (4\eta)^2 (L_0 + L_2 \delta m^2 + L_4 \delta m^4 + [\eta J_{\text{ISCO}} / m^2] \delta m^6), \]

With \( L_0, L_2, L_4 \), fitting parameters.

\[ L_{\text{peak}}, \omega_{\text{peak}}, h_{\text{peak}}, \text{etc all follow analogous expansions to } M_{\text{final}}. \]
The Non-spinning case Revisited

A. Recoil velocities of non-spinning binaries

Consistent with our notation in Ref. [14], we expand the non-spinning recoil as

\[ v_m = \eta^2 \delta m \left( A + B \delta m^2 + C \delta m^4 \right) \]  

(1)

where \( \delta m = (m_1 - m_2)/m \) and \( m = (m_1 + m_2) \) and

\[ 4\eta = 1 - \delta m^2. \]

TABLE V. Fitting to the recoil velocity of the remnant of nonspinning black hole binaries by Eq. (1). Fit 2 only uses \( A, B \) while Fit 3 also fits \( C \). Standard error for each fit is also given.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit [42]</th>
<th>Fit 2</th>
<th>Fit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>-9210</td>
<td>-8919 ± 73</td>
<td>-8712 ± 32</td>
</tr>
<tr>
<td>( B )</td>
<td>-2790</td>
<td>-4273 ± 261</td>
<td>-6516 ± 256</td>
</tr>
<tr>
<td>( C )</td>
<td>0.0</td>
<td>0.0</td>
<td>3907 ± 424</td>
</tr>
</tbody>
</table>


FIG. 1. Our current fit and the original González et al. [42] fit to the recoils from nonspinning BHBs. The panel below gives the residual and percent difference of both fits.
B. Peak luminosity of non-Spinning Binaries

\[ L_{\text{peak}} = (4\eta)^2 \left\{ N_0 + N_{2d} \delta m^2 + N_{4f} \delta m^4 \right\} \]  \hspace{1cm} (2)

Note that the radiated power in the particle limit scales as \( \eta^2 \) [see Ref. [43], Eq. (16) and (20); evaluated at the ISCO for its peak value].


<table>
<thead>
<tr>
<th>Parameter</th>
<th>( L_{\text{peak}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_0 )</td>
<td>( 1.026 \times 10^{-3} \pm 1.727 \times 10^{-6} )</td>
</tr>
<tr>
<td>( N_{2d} )</td>
<td>( -4.092 \times 10^{-4} \pm 2.847 \times 10^{-5} )</td>
</tr>
<tr>
<td>( N_{4f} )</td>
<td>( 2.422 \times 10^{-4} \pm 6.552 \times 10^{-5} )</td>
</tr>
</tbody>
</table>
C. Peak frequency and amplitude of non-Spinning Binaries

Analogously to the previous formula to model the peak luminosity, we introduce the following fitting formula for the peak frequency of the (2, 2) mode of the gravitational wave strain for nonspinning binaries

\[ m \omega_{22}^{\text{peak}} = \left\{ W_0 + W_2 \delta m^2 + W_4 \delta m^4 \right\}, \tag{3} \]

The results of fitting the parameters \( W_0, W_2, \) and \( W_4 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit 1</th>
<th>Parameter</th>
<th>Fit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_0 )</td>
<td>0.3587 ± 0.0008</td>
<td>( W_0' )</td>
<td>0.3580 ± 0.0010</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>-0.1211 ± 0.0036</td>
<td>( W_2' )</td>
<td>0.2466 ± 0.0093</td>
</tr>
<tr>
<td>( W_4 )</td>
<td>0.0432 ± 0.0034</td>
<td>( W_4' )</td>
<td>0.2718 ± 0.0129</td>
</tr>
</tbody>
</table>

If we impose the particle limit peak frequency, \( m_f \Omega_p = 0.2795 \) into our formula, we have the alternative Fit 2:

\[ m \omega_{22}^{\text{peak}} = (4\eta) \left\{ W_0' + W_2' \delta m^2 + W_4' \delta m^4 \right\} + m_f \Omega_p \delta m^6, \tag{4} \]

where \( \eta = (1 - \delta m^2)/4. \)


FIG. 3. Current fit and the Bohé et al. [44] fit to the peak waveform frequency from nonspinning BHBs.
The particle limit represents a test and a potential improvement for remnant formulae.

- If we have an expression for the particle limit we can readily and effectively incorporate in our formulae with clear benefits, as for $m_{\text{final}}$ and $S_{\text{final}}$.
- NR simulations can also accurately extrapolate from the comparable mass limit and make predictions of what the perturbative, semi-analytic computations should produce.
- Concrete examples of high accurate computations are the final mass and spin, but the computation of recoil velocities, peak luminosity and peak frequencies would highly benefit of perturbative computations.
- In particular, the peak frequency can take also an additional $\eta$–correction.
- Should we rerun $q=1/100$?
- A predicted $h_{\text{peak}} - \omega_{\text{peak}}$ can be used as a test of GR.
Additional material on the accuracy of the simulations regarding the three main sources of error:

Finite Extraction Radius

Finite Resolution

Finite sum over modes

From appendices in


FIG. 8. Plots of the convergence of the recoil velocity (top left), peak luminosity (top right), energy radiated (bottom left), and angular momentum radiated (bottom right) as a function of $m/r_{\text{obs}}$ for case 80—Q1.0000_-0.8000_0.8000. Horizontal green solid lines in the bottom row indicate the energy and angular momentum radiated calculated from the isolated horizon. The dark gray lines in each plot shows the extrapolation to infinite observer location using only up to $r = 113$ m.
FIG. 9. Plots of the convergence of the recoil velocity (top left), peak luminosity (top right), energy radiated (bottom left), and angular momentum radiated (bottom right) as a function of $m/r_{\text{obs}}$ for case 47—Q0.7500_-0.8000_0.8000. Horizontal green solid lines in the bottom row indicate the energy and angular momentum radiated calculated from the isolated horizon.
FIG. 10. Plots of the convergence of the recoil velocity (top left), peak luminosity (top right), energy radiated (bottom left), and angular momentum radiated (bottom right) as a function of $m/r_{\text{obs}}$ for case 67—Q0.5000_0.0000_0.0000. Horizontal green solid lines in the bottom row indicate the energy and angular momentum radiated calculated from the isolated horizon.
FIG. 11. Plots of the convergence of the recoil velocity (top left), peak luminosity (top right), energy radiated (bottom left), and angular momentum radiated (bottom right) as a function of $m/r_{\text{obs}}$ for case 65—Q0.3333_0.0000_0.0000. Horizontal green solid lines in the bottom row indicate the energy and angular momentum radiated calculated from the isolated horizon.
FIG. 22: The recoil velocity as computed at a given extraction radius: $75M - 190M$ and extrapolations to infinity. The different curves correspond to the two initial separations labeled as A and B and as a function of resolution (Low - Medium - High) refined by a global factor 1.2. A quadratic least squares fit is shown for each.

FIG. 23: The dependence of the computed recoil velocity on the number of $\ell$ modes used to construct the radiated linear momentum. Here all modes with $\ell \leq \ell_{\text{max}}$ were used and we show the recoil for the A and B configurations for the Low, Medium, and High resolution runs.

FIG. 24: Above: The radiated energy as computed at a given extraction radius: $75M - 190M$ and extrapolations to infinity. The different curves correspond to the two initial separations labeled as A and B and as a function of resolution (Low - Medium - High) refined by a global factor 1.2. A quadratic least squares fit is shown for each. Below: The dependence of the computed radiated energy on the number of $l$ modes used to construct it. Here all modes with $l \leq l_{\text{max}}$ were used. The black and gray lines labeled with "IH" are the associated final mass calculated from the BH horizon. On this scale, all resolutions are on top of one another, so only one line is shown.

FIG. 25: Above: The radiated angular momentum as computed at a given extraction radius: $75M - 190M$ and extrapolations to infinity. The different curves correspond to the two initial separations labeled as A and B and as a function of resolution (Low - Medium - High) refined by a global factor 1.2. A quadratic least squares fit is shown for each. Below: The dependence of the computed radiated angular momentum on the number of $l$ modes used to construct it. Here all modes with $l \leq l_{\text{max}}$ were used. The black and gray lines labeled with "IH" are the associated final spin calculated from the BH horizon. On this scale, all resolutions are on top of one another, so only one line is shown.